Closed-Form Expressions for the Design of Microstrip Lines with Two Substrate Layers *

James P. K. Gilb and Constantine A. Balanis

Department of Electrical Engineering
Telecommunications Research Center
Arizona State University
Tempe, Arizona 85287-7206

ABSTRACT

A simple closed-form expression for the effective dielectric constant of a microstrip on a two-layer substrate is presented. Results for single layer cases are incorporated in the formula to simplify it and increase the accuracy. The formula is curve-fitted from results computed with the Spectral Domain approach and is accurate to within three percent relative error over a wide range of parameters.

INTRODUCTION

As more microwave circuit applications move into the commercial field, considerations such as process yield, repeatability and manufacturability place additional demands on microwave design tools. While full-wave finite-element and moment-method *analysis* tools are now becoming available that provide very accurate results, *design* tools must trade some accuracy for speed, returning very good results in minutes, instead of hours. These design tools must be based on an accurate full-wave technique so that the designed element has the desired performance when used in the analysis tool. For example, a transmission line that is 50Ω in the design tool should not evaluate to a 45Ω line on the analysis tool. To analyze the process yield, many similar cases must be analyzed to calculate the sensitivity of the design to process variation, emphasizing the need for very fast design tools. Complicating the microwave design problem is the increased use of multi-layer geometries, for which there are no accurate formulas available. Accurate and efficient design tools that can be used for multi-conductor, multi-layer geometries are required to achieve the goals of successful microwave design.

Accurate closed-form expressions for the transmission line parameters, ϵ_{reff} and Z_0, of open microstrips on a single substrate have been available for many years [1], [2]. Closed-form expressions for the transmission line parameters of coupled microstrips on suspended substrate have been made available more recently [3]. However, there are not any closed-form expressions for the parameters of open microstrips in more general structures, even for the simple single-conductor, two-layer substrate case. The goal of this research is to develop closed-form expressions for the design of open microstrip lines with two substrate layers.

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ANALYSIS

The general trends of the parameters of a single substrate microstrip are well-known, e.g. wider center conductors have increased ϵ_{reff} and decreased impedance. However, for as few as two substrate layers, the general trends of the transmission line parameters become much more complicated. For example, if the dielectric constant of the upper substrate is much greater than that of the lower substrate, the ϵ_{reff} for the odd mode can be higher than that of the even mode [4], which is the reverse of the single layer case. Furthermore, increasing the width of the center conductor can actually decrease the effective dielectric constant for a single microstrip on a two-layer substrate. Thus it is important that any closed-form expression accurately take into account the unique behavior of multiple substrate structures.

The design tools proposed in this work have been generated using the Spectral Domain Approach [5] to compute the transmission line parameters of single and coupled microstrips on two-layer substrates. A recurrence relation is used to compute the dyadic Green's function [4] and the expansion functions chosen are Chebyshev polynomials modifying the appropriate edge conditions [6]. Five longitudinal and four transverse current expansion functions are used to insure accurate results, with all calculations being done in double precision. The analysis is valid in the quasi-static region. The numerical results from the SDA are used to create curve-fitted formulas that are valid for a wide range of substrate parameters and conductor configurations. Wherever possible, the results have been compared to previously

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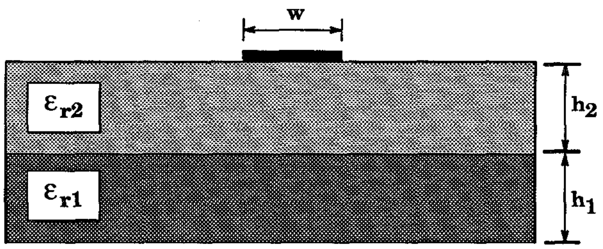

Figure 1: Geometry of coupled microstrips with two substrate layers.

published data and other formulas to verify the accuracy of these new formulas.

The geometry of the two-substrate layer microstrip used in the present analysis is shown in Fig. 1. The total height of the substrate layers, $h_{tot} = h_1 + h_2$, is held constant and the design formula uses the following as design parameters:

- ratio of the width to total height, w/h_{tot}
- the height ratio, $h_r = h_1/h_{tot}$
- the dielectric constants, ϵ_{r1} and ϵ_{r2}
- the effective dielectric constants of the single layer cases, $\epsilon_{reff}(h_r = 0)$ and $\epsilon_{reff}(h_r = 1)$

We seek a formula that relates the above parameters of the substates to the transmission line parameters of this structure.

Fig. 2 through Fig. 4 show the effective dielectric constant as a function of the the height ratio for different dielectric constants of the lower substate in the quasi-static region. Three different normalization schemes are used in the graphs. In Fig. 2, a linear normalization is used, described by

$$\epsilon_{reff(linear)}(h_r) = \epsilon_{reff}(h_r) - \epsilon_{reff}(0) - h_r\left[\epsilon_{reff}(1) - \epsilon_{reff}(0)\right] \quad (1)$$

where $\epsilon_{reff}(0)$ and $\epsilon_{reff}(1)$ are the effecitive dielectric constants for the single substrate cases, given by

$$\epsilon_{reff}(0) = \epsilon_{reff}\big|_{h_1/h_{tot}=0.0} \quad (2)$$

$$\epsilon_{reff}(1) = \epsilon_{reff}\big|_{h_1/h_{tot}=1.0} \quad (3)$$

These values can be determined from any of the many approximate formulas for the effective dielectric constant of a single substrate structure. While this normalization gives reasonable results when $\epsilon_{r1} > \epsilon_{r2}$, when $\epsilon_{r1} < \epsilon_{r2}$ the results are functions that are much more complex. In addition, this normalization does not result in a fit that is monotonic as a function of the substrate dielectric constant.

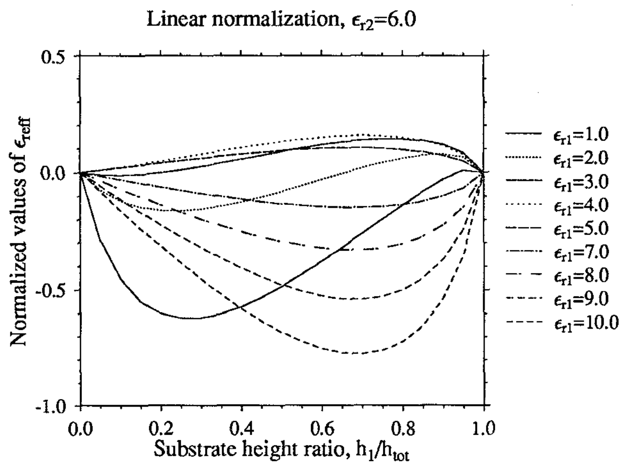

Figure 2: ϵ_{reff} of a microstrip on a two-layer substrate normalized using (1) as a function of the substrate height ratio for different values of ϵ_{r1} ($w/h_{tot} = 1.0$).

In Fig. 3, the normalization is based on the capacitance of a parallel plate waveguide with two dielectric layers and is given by

$$\epsilon_{reff(ppwg)}(h_r) = \frac{\epsilon_{reff}(0)\epsilon_{reff}(1)}{h_r\left[\epsilon_{reff}(0) - \epsilon_{reff}(1)\right] + \epsilon_{reff}(1)} - \epsilon_{reff}(h_r) \quad (4)$$

Using this normalization gives a better initial approximation to the effective dielectric constant and makes the graphs more uniform for the case when $\epsilon_{r1} < \epsilon_{r2}$. In addition, the normalized functions are monotonic as a function of the substrate dielectric constants, simplifying the curve-fitting procedure.

The final normalization, shown in Fig. 4 involves mapping the height ratio such that preceeding equation becomes a linear one. The equation for remapping the height ratio, and the corresponding normalization for ϵ_{reff} are given by

$$h_r' = \left\{\frac{\epsilon_{reff}(0)\epsilon_{reff}(1)}{h_r\left[\epsilon_{reff}(0) - \epsilon_{reff}(1)\right] + \epsilon_{reff}(1)} - \epsilon_{reff}(0)\right\} \cdot \frac{1}{\epsilon_{reff}(1) - \epsilon_{reff}(0)} \quad (5)$$

$$\epsilon_{reff(ppwg)}(h_r) = \epsilon_{reff}(h_r) + \epsilon_{reff}(0) - h_r'\left[\epsilon_{reff}(1) - \epsilon_{reff}(0)\right] \quad (6)$$

This mapping is the one most suited to curve fitting the data, giving an overall relative error that is lower than the error obtained with other techniques using the same order of fitting function.

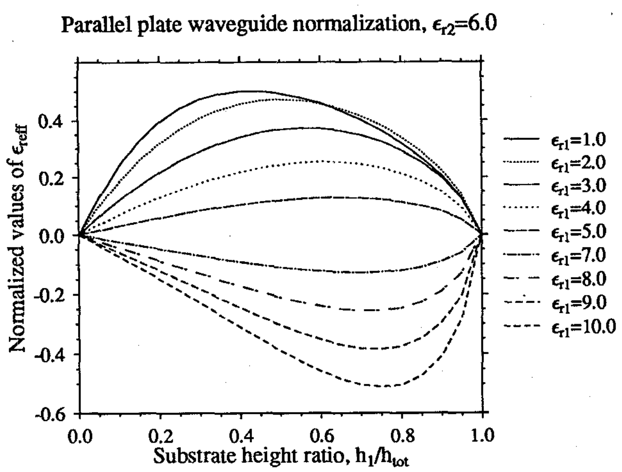

Figure 3: ϵ_{reff} of microstrip on a two-layer substrate normalized using (4) as a function of the substrate height ratio for different values of ϵ_{r1} $(w/h_{tot} = 1.0)$.

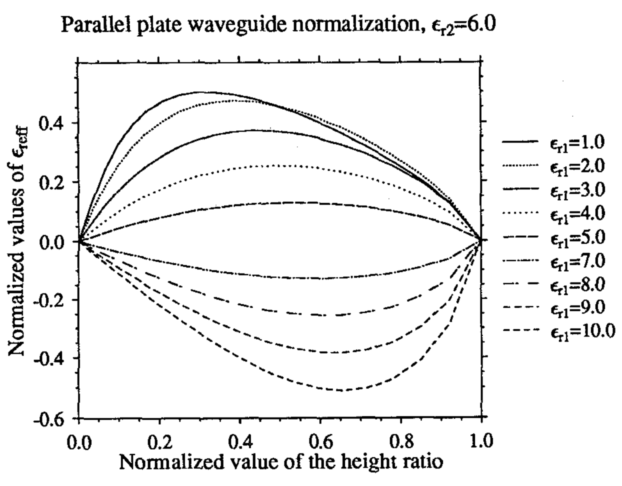

Figure 4: ϵ_{reff} of microstrip on a two-layer substrate normalized using (6) as a function of the substrate height ratio normalized using (5) for different values of ϵ_{r1} $(w/h_{tot} = 1.0)$.

The full formula for a single microstrip is given by

$$\begin{aligned}\epsilon_{\text{reff}}(h_r) &= \frac{\epsilon_{\text{reff}}(0)\epsilon_{\text{reff}}(1)}{h_r\left[\epsilon_{\text{reff}}(0)-\epsilon_{\text{reff}}(1)\right]+\epsilon_{\text{reff}}(1)} \\ &\quad + \left\{\sum_{n=1}^{5} b_n T_n(h_r')^n + \sum_{n=1}^{2} a_n(\epsilon_{rn}-1)\right\} \\ &\quad \cdot h_r'(1-h_r') \qquad (7) \\ &= h_r'\left[\epsilon_{\text{reff}}(1)-\epsilon_{\text{reff}}(0)\right]+\epsilon_{\text{reff}}(0) \\ &\quad + \left\{\sum_{n=1}^{5} b_n T_n(h_r')^n + \sum_{n=1}^{2} a_n(\epsilon_{rn}-1)\right\} \\ &\quad \cdot h_r'(1-h_r') \qquad (8)\end{aligned}$$

where $T_n(h_r')$ is the Chebyshev polynomial of the first kind of order n. The coefficients in the above expression, b_n and a_n, can be determined using the following relations;

$$b_n = \left[\epsilon_{\text{reff}}(1)-\epsilon_{\text{reff}}(0)\right]\sum_{i=1}^{4}\sum_{j=1}^{5-i} c_{nij}\epsilon_{\text{reff}}^{i}(1)\epsilon_{\text{reff}}^{j}(0) \qquad (9)$$

$$a_n = \sum_{i=1}^{4} d_{ni}(w/h_{tot}-1.0)^i \qquad (10)$$

where the constants c_{nij} and d_{ni} are given in Table 1 and Table 2. Most of the dependence on w/h_{tot} is implicitly contained in $\epsilon_{\text{reff}}(0)$ and $\epsilon_{\text{reff}}(1)$ and so only a small, and simple correction factor is used to improve the accuracy of the solution over a wider range of width to height ratios. The relative error of the function is defined here as

$$(\text{error})_{relative} = 1 - \frac{\epsilon^{*}_{\text{reff}}(h_r, w/h_{tot}, \epsilon_{r1}, \epsilon_{r2})}{\epsilon_{\text{reff}}(h_r, w/h_{tot}, \epsilon_{r1}, \epsilon_{r2})} \qquad (11)$$

where $\epsilon^{*}_{\text{reff}}$ is the approximate value calculated by (8) and ϵ_{reff} is the value calculated by the SDA. For $0.2 \le w/h_{tot} \le 3.5$, (8) gives results that are accurate to within 3 percent relative error over all values of the height ratio and for $1 \le \epsilon_{r1}, \epsilon_{r2} \le 10$.

CONCLUSIONS

A closed-form expression for the effective dielectric constant of an open microstrip on a two-layer substrate was presented that gives results that have less than three percent relative error. The formula uses the value of the effective dielectric constant of two single layer cases in addition to the physical parameters to provide the accurate results. Results have been validated in the quasi-static region with $0.2 \le w/h_{tot} \le 3.5$, $1 \le \epsilon_{r1}, \epsilon_{r2} \le 10$ and for all values of the height ratio.

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c_{1ij}				
j	i=1	i=2	i=3	i=4
1	-1.043209	-0.0503390	-0.0797488	0.00798507
2	-0.675972	0.123333	-0.00725776	
3	0.115952	-0.00501308		
4	-0.00759135			

c_{2ij}				
j	i=1	i=2	i=3	i=4
1	-0.146779	0.474085	-0.0278816	0.00152566
2	0.539591	-0.142693	0.00435987	
3	-0.103164	0.0104707		
4	0.00659505			

c_{3ij}				
j	i=1	i=2	i=3	i=4
1	0.427940	-0.976178	0.0920436	-0.00530197
2	-0.538552	0.235510	-0.00943642	
3	0.0863343	-0.0152405		
4	-0.00494465			

c_{4ij}				
j	i=1	i=2	i=3	i=4
1	-0.04520958	0.388317	-0.0176737	7.35419×10^{-4}
2	0.109800	-0.107298	0.00282642	
3	-0.00953375	0.00784613		
4	-4.78285×10^{-5}			

c_{5ij}				
j	i=1	i=2	i=3	i=4
1	0.0749343	-0.233050	0.0164757	-8.25541×10^{-4}
2	-0.0811032	0.0594164	-0.00200066	
3	0.00974942	-0.00406101		
4	-3.52724×10^{-4}			

Table 1: Coeficients, c_{nij}, for (9)

i	n=1	n=2
1	1.94409×10^{-2}	-5.00773×10^{-3}
2	-7.09419×10^{-3}	4.91136×10^{-3}
3	2.41479×10^{-3}	-2.12875×10^{-3}
4	-3.92206×10^{-4}	3.55627×10^{-3}

Table 2: Coeficients, d_{ni}, for (9)

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